

WHAT IS CLAIMED:

1. A method for predicting pressure independent permeation flux and target molecule yield in a permeate resulting from crossflow membrane filtration of particles in a poly-disperse suspension, said method comprising:
 - determining particle size distribution of the poly-disperse suspension;
 - determining equivalent spherical radii of the particles;
 - determining viscosity of the suspension;
 - determining maximum back-transport velocity (u_i) for all particles;
 - estimating maximum aggregate packing volume fraction (ϕ_M) for all particles at a wall of the filtration membrane from geometric considerations;
 - selecting the particle that gives a minimum permeation flux at a given filtration membrane shear rate, wherein the selected particle has a radius (a_i);
 - determining a predicted permeation flux (J);
 - determining packing density ϕ_{wi} at a membrane wall for each particle size (a_j for $j \neq i$) at the predicted permeation flux;
 - determining interstitial packing density ($\phi_{wiinterstice}$) of particles in the suspension which are the smallest;
 - determining minimum pore diameter ($2r_{\text{minimum}}$) based on the packing density of each particle; and
 - estimating yield of a target species in the filtration permeate by calculating observed sieving coefficient (S_o) for the target species, thereby predicting permeation flux and target molecule yield of the poly-disperse suspension during crossflow filtration.
2. The method according to claim 1, wherein said determining viscosity of the suspension is carried out by using a modified Einstein-Smoluchowski equation: $\eta/\eta_0 = 1 + 2.5\phi_b + k_1\phi_b^2$, where η is bulk fluid viscosity (kg/m.s) of the suspension, η_0 is bulk fluid viscosity of the suspension without

solute (kg/m.s), k_1 is particle shape factor (-), and ϕ_b is particle volume fraction in the bulk suspension (-).

3. The method according to claim 1, wherein said determining viscosity of the suspension is carried out by experiment.

4. The method according to claim 1, wherein said determining maximum back-transport velocity (u_i) comprises:

calculating Brownian diffusion (J_B) for all particles, where

$$J_B = 0.114(\gamma\kappa^2 T^2 / \eta^2 a^2 L)^{1/3} \ln(\phi_w / \phi_b);$$

calculating inertial lift (J_I) for all particles, where $J_I =$

$$0.036 \rho a^3 \gamma^2 / \eta;$$

calculating shear induced diffusion (J_S) for all particles where $J_S = 0.078(a^4/L)^{1/3} \gamma \ln(\phi_w / \phi_b)$, wherein γ is wall shear rate (s^{-1}), κ is Boltzmann constant (J/mol K), T is temperature (K), η is bulk fluid viscosity (kg/m.s), a_i is radius of species i (m), L is tube length (m), ϕ_w is particle volume fraction at the filtration membrane (-), ϕ_b is the particle volume fraction in the bulk suspension (-), and ρ is particle density (kg/m^3); and

selecting J_{max} for each particle, wherein $J_{max} = u_i$, whereby maximum back-transport for each particle is determined.

5. The method according to claim 1, wherein said estimating maximum aggregate packing volume fraction (ϕ_M) at the membrane wall for a suspension comprises:

determining particle size (a_i) of species (i) in the suspension;

determining if the size ratio of the particles is > 10 , such that $a_{i+1} > 10a_i$ for all a_i ; and

calculating the maximum aggregate packing volume fraction (ϕ_{Mn}) by $\phi_{Mn} = \phi_m + \phi_m (1 - \phi_{Mn-1})$, where $\phi_M = \phi_m$ is set to 0.64 when the size ratio of the particles is > 10 , such that $a_{i+1} > 10a_i$ for all a_i .

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6. The method according to claim 5, wherein the suspension comprises 3 particle sizes and wherein $a_1 > 10a_2 > 100a_3$, said method further comprising:

calculating $\phi_M = \phi_m + \phi_m (1 - \phi_m) + 0.74[1 - \{\phi_m + \phi_m (1 - \phi_m)\}]$,

wherein ϕ_m is set to 0.64.

7. The method according to claim 1, wherein said estimating maximum aggregate packing volume fraction (ϕ_M) at the membrane wall for a suspension comprising two particles, such that $a_1 > 10 a_2$, is carried out by calculating $\phi_M = \phi_m + 0.74 (1 - \phi_m)$, where ϕ_m is set to 0.64.

8. The method according to claim 1, wherein said estimating maximum aggregate packing volume fraction (ϕ_M) at the membrane wall comprises:

calculating a maximum radius ratio of all particles;

determining if said maximum radius ratio is < 10 ; and

setting ϕ_M as 0.68, where said maximum radius ratio is < 10 .

9. The method according to claim 1, wherein said selecting the particle that gives the minimum permeation flux (J) comprises:

calculating Brownian diffusion (J_B) for all particles, where

$$J_B = 0.114(\gamma\kappa^2 T^2 / \eta^2 a^2 L)^{1/3} \ln(\phi_w / \phi_b);$$

calculating inertial lift (J_I) for all particles, where $J_I =$

$$0.036\rho a^3 \gamma^2 / \eta;$$

calculating shear induced diffusion (J_S) for all particles, where $J_S = 0.078(a^4/L)^{1/3} \gamma \ln(\phi_w / \phi_b)$, wherein γ is wall shear rate (s^{-1}), κ is Boltzmann constant (J/mol K), T is temperature (K), η is bulk fluid viscosity (kg/m.s), a_i is radius of species i (m), L is tube length (m), ϕ_w is particle volume fraction at the membrane wall (-), ϕ_b is the particle volume fraction in the bulk suspension (-), and ρ is particle density (kg/m^3);

determining a J_{max} value for each particle; and

selecting a J_{max} value from among all J_{max} values that is the lowest, thereby selecting the minimum permeation flux (J).

10. The method according to claim 1, wherein said determining packing density at the membrane wall (ϕ_{wj}) for all particles at the predicted permeation flux (a_j for $j \neq i$) comprises:

back-calculating the value of ϕ_{wj} such that ϕ_{wj} gives the predicted permeation flux (J) of selected particle (a_i) using the equation for back-transport that establishes maximum back transport for each particle (a_j for $j=i$), wherein the equation is either $J_B = 0.114(\gamma\kappa^2T^2/\eta^2a^2L)^{1/3}\ln(\phi_w/\phi_b)$ or $J_S = 0.078(a^4/L)^{1/3}\gamma\ln(\phi_w/\phi_b)$, or $J_I = 0.036\rho a^3\gamma^2/\eta$, where γ is wall shear rate (s^{-1}), κ is Boltzmann constant ($J/mol\ K$), T is temperature (K), η is bulk fluid viscosity (kg/m.s), a_i is radius of species i (m), L is tube length (m), ϕ_w is particle volume fraction at the membrane wall (-), ϕ_b is the particle volume fraction in the bulk suspension (-), and ρ is particle density (kg/m^3).

11. The method according to claim 10, wherein said determining packing density further comprises:

determining if the predicted permeation flux is established by inertial lift (J_I) for one particle type;

determining if $u_{f1} \geq 10J$; and

setting $\phi_{wj} = 0$, when one particle type is established by inertial lift (J_I) and $u_{f1} \geq 10J$.

12. The method according to claim 10, wherein said determining packing density further comprises:

determining if the predicted permeation flux is established by inertial lift (J_I) for one particle type;

determining if $u_{f1} < 10J$; and

determining packing density (ϕ_{wj}) by $\phi_{wj1} = \phi_M - \sum \phi_{wj}$ when $u_{f1} < 10J$ and one particle type is established by inertial lift.

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13. The method according to claim 10, wherein said determining packing density further comprises:

- determining if permeation flux is established by inertial lift (J_1) for more than one particle type;
- determining if $u_{j1} < 10J$ for the particles; and
- determining packing density by $\phi_{wj1} = \phi_M - \sum \phi_{wj}$ when $u_{j1} < 10J$ and permeation flux is established by inertial lift for more than one particle type.

14. The method according to claim 10, wherein said determining packing density further comprises:

- determining if permeation flux is established by J_1 for more than one particle type ($j11, j12, \dots, j1n$); and
- determining packing density at the membrane wall by $\phi_{wj11} + \phi_{wj12} = \phi_M - \sum \phi_{wj}$, wherein $\phi_{wj11} : \phi_{wj12} = \phi_{bj11} u_{j12} : \phi_{bj12} u_{j11}$, where $j \neq j11$ or $j12$ and $u_{j11}, u_{j12} < 10J$, when permeation flux is established by J_1 for more than one particle type.

15. The method according to claim 1, wherein said determining interstitial packing density ($\phi_{wiinterstice}$) of the smallest particle is carried out by $\phi_{wiinterstice} = \phi_{wicorrected} / (1 - \sum \phi_{wjcorrected})$, wherein $\phi_{wicorrected} = \phi_M [(\phi_{wi}) / \sum \phi_{wi}]$, where ϕ_{wi} is the particle volume fraction at the membrane wall (-) for particle i .

16. The method according to claim 1, wherein said determining minimum pore diameter ($2r_{\text{minimum}}$) is carried out using $2r_{\text{minimum}} = a_i \{ \sqrt[3]{2[4(4/3)\pi/\phi_{wiinterstice}] - 2} \}$, where a is radius of species i (m) and r_{minimum} is a minimum equivalent cake void radius for all cake types (m).

17. The method according to claim 1, wherein said estimating yield of a target species comprises:

- calculating observed sieving coefficient (S_o), where $S_o = S_a / ((1 - S_a) \exp(-J/k) + S_a)$, wherein actual sieving coefficient S_a is obtained from

$S_a = (S_\infty \exp(Pe_m)) / (S_\infty + \exp(Pe_m) - 1)$, wall Peclet number, Pe_m is obtained from $Pe_m = (J\delta_m/D)(S_\infty / \varepsilon\phi K_d)$, where J is permeation flux (m/s), δ_m is taken as the side of the face centered cube of the particles of radius a_i that forms the controlling cake for transmission, $\delta_m = a = a_i [(4(4/3)\pi)/\phi_{interstice}]^{1/3}$, D is molecular diffusion coefficient (m²/s), intrinsic sieving coefficient S_∞ is obtained from $S_\infty = (1-\lambda)^2 [2 - (1-\lambda)^2] \exp(-0.7146\lambda^2)$, $\lambda = r_s/r_{min}$, where r_s is solute radius (m) and r_{min} is a minimum equivalent cake void radius for all cake types (m), ϕ is equilibrium partition coefficient between membrane pore and suspension (-), ε is cake/membrane porosity (-), K_d is hindrance factor for diffusive transport (-), and k is mass transfer coefficient (m/s).

18. The method according to claim 1, wherein crossflow-filtration is carried out in a diafiltration mode, and the yield of the target species is estimated after N_d diavolumes as $\text{Yield} = 1 - \exp(-N_d S_{o\text{average}})$ where $S_{o\text{average}}$ is average observed sieving coefficient during diafiltration (-), where $S_o = S_a / ((1 - S_a) \exp(-J/k) + S_a)$, where actual sieving coefficient S_a is obtained from $S_a = (S_\infty \exp(Pe_m)) / (S_\infty + \exp(Pe_m) - 1)$, where J is permeation flux (m/s), wall Peclet number, Pe_m , is obtained from $Pe_m = (J\delta_m/D)(S_\infty / \varepsilon\phi K_d)$, where δ_m is taken as the side of the face centered cube of the particles of radius a_i that forms the controlling cake for transmission, where $\delta_m = a = a_i [(4(4/3)\pi)/\phi_{interstice}]^{1/3}$, D is molecular diffusion coefficient (m²/s), intrinsic sieving coefficient S_∞ is obtained from $S_\infty = (1-\lambda)^2 [2 - (1-\lambda)^2] \exp(-0.7146\lambda^2)$, $\lambda = r_s/r_{min}$, where r_s is solute radius (m) and r_{min} is a minimum equivalent cake void radius for all cake types (m), ϕ is equilibrium partition coefficient between membrane pore and suspension (-), ε is cake/membrane porosity (-), K_d is hindrance factor for diffusive transport (-), and k is mass transfer coefficient (m/s).

19. The method according to claim 1 further comprising:
re-calculating packing density for all particle sizes if packing constraints are not satisfied based on initial determination of packing densities of the particles at the wall.

20. The method according to claim 19 further comprising:
correcting packing density using $\phi_{w\text{icorrected}} = \phi_M [(\phi_{wi})/\Sigma \phi_{wi}]$;
reevaluating J for the particle selected as having the minimum
permeation flux based on $\phi_{w\text{icorrected}} = \phi_M [(\phi_{wi})/\Sigma \phi_{wi}]$; and
reevaluating maximum back-transport velocity (u_i).
21. The method according to claim 20 further comprising:
repeating the steps of claim 17 until a desired packing constraint is
met.
22. The method according to claim 1 further comprising:
refining the yield of the target species.
23. The method according to claim 22, wherein said refining
the yield comprises:
determining whether the suspension has a low, intermediate, or
high operating shear rate leading to different yield regimes, wherein a suspension
at a low operating shear rate leads to an $S_o \geq 0.75$ corresponding to a yield ≥ 0.95 ,
an intermediate operating shear rate leads to $0 < S_o < 0.75$ corresponding to yield
from 0 to 95%, or a high operating shear rate leads to an $S_o \cong 0$, wherein $S_o =$
 $S_a / ((1 - S_a) \exp(-J/k) + S_a)$, wherein actual sieving coefficient S_a is obtained from S_a
 $= (S_\infty \exp(Pe_m)) / (S_\infty + \exp(Pe_m) - 1)$, wall Peclet number, Pe_m is obtained from
 $Pe_m = (J \delta_m / D)(S_\infty / \varepsilon \phi K_d)$, where J is permeation flux (m/s), δ_m is taken as the side
of the face centered cube of the particles of radius a_i that forms the controlling
cake for transmission, $\delta_m = a = a_i [(4(4/3)\pi) / \phi_{\text{interstice}}]^{1/3}$, D is molecular diffusion
coefficient (m²/s), intrinsic sieving coefficient S_∞ is obtained from $S_\infty = (1 - \lambda)^2 [2 -$
 $(1 - \lambda)^2] \exp(-0.7146\lambda^2)$, $\lambda = r_s / r_{\text{min}}$, where r_s is solute radius (m) and r_{min} is a
minimum equivalent cake void radius for all cake types (m), ϕ is equilibrium
partition coefficient between membrane pore and suspension (-), ε is
cake/membrane porosity (-), K_d is hindrance factor for diffusive transport (-), and
 k is mass transfer coefficient (m/s).

24. The method according to claim 23, wherein an intermediate operating shear rate is determined as leading to $0 < S_o < 0.75$, said method further comprising:

calculating stagnant film flux (J) equation for non-retentive membranes wherein $J = k \ln [(\phi_{wi} - \phi_{permeatei}) / (\phi_{bi} - \phi_{permeatei})] \cong k \ln [\phi_{wi} / \phi_{bi}(1 - S_o)]$, wherein $(\phi_{wi} \gg \phi_{permeatei})$; and

correcting S_o by replacing $J =$ solvent permeation flux (m/s) with the stagnant film flux (J) equation for non-retentive membranes in the equation for observing sieving coefficient, S_o , where $S_o = S_a / ((1 - S_a) \exp(-J/k) + S_a)$.

25. The method according to claim 1 further comprising:
constructing a plot of the predicted permeation flux and yield versus wall shear rate, thereby predicting permeation flux and target molecule yield of the poly-disperse suspension during microfiltration.

26. The method according to claim 1, wherein filtration is selected from the group consisting of microfiltration and ultrafiltration.

27. The method according to claim 1, wherein filtration is carried out with a filter selected from the group consisting of a flat sheet filter, hollow-fiber filter, and a helical filter.

28. The method according to claim 1, wherein the suspension is selected from the group consisting of streams from biomedical and bio-processing industries, waste water, surface water, environmental pollutants, industrial waste streams, and industrial feed streams.

29. The method according to claim 28, wherein the suspension is a stream from biomedical and bio-processing industries selected from the group consisting of proteins, cells, nucleic acids, colloids, milk, and suspended particles.

30. A method for determining packing density of particles of a poly-disperse suspension at a membrane wall, said method comprising:

providing a predicted permeation flux (J);

determining packing density for all particle sizes at the predicted permeation flux; and

determining interstitial packing density ($\phi_{wiinterstice}$) of particles in the suspension which are smallest, thereby determining packing density at the membrane wall of particles of the poly-disperse suspension.

31. The method according to claim 30, wherein said determining packing density at the membrane wall (ϕ_{wj}) for all other particles at the predicted permeation flux (a_j for $j \neq i$) comprises:

back-calculating the value of ϕ_{wj} such that ϕ_{wj} gives the predicted permeation flux (J) of selected particle (a_i), using the equation for back-transport that establishes maximum back transport for each particle (a_j for $j=i$), wherein the equation is either $J_B = 0.114(\gamma\kappa^2T^2/\eta^2a^2L)^{1/3}\ln(\phi_w/\phi_b)$ or $J_S = 0.078(a^4/L)^{1/3}\gamma\ln(\phi_w/\phi_b)$, or $J_I = 0.036\rho a^3\gamma^2/\eta$, where γ is wall shear rate (s^{-1}), κ is Boltzmann constant ($J/mol K$), T is temperature (K), η is bulk fluid viscosity (kg/m.s), a_i is radius of species i (m), L is tube length (m), ϕ_w is particle volume fraction at the membrane wall (-), ϕ_b is the particle volume fraction in the bulk suspension (-), and ρ is particle density (kg/m³).

32. The method according to claim 31, wherein said determining packing density further comprises:

determining if the predicted permeation flux is established by inertial lift (J_I) for one particle type;

determining if $u_{fl} \geq 10J$; and

setting $\phi_{wj} = 0$, when one particle type is established by inertial lift (J_I).

33. The method according to claim 31, wherein said determining packing density further comprises:

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determining if the predicted permeation flux is established by inertial lift (J_I) for one particle type;
determining if $u_{jI} < 10J$; and
determining packing density (ϕ_{wj}) by $\phi_{wjI} = \phi_M - \sum \phi_w$, when $u_{jI} < 10J$ and one particle type is established by inertial lift.

34. The method according to claim 31, wherein said determining packing density further comprises:

determining if permeation flux is established by inertial lift (J_I) for more than one particle type;
determining if $u_{jI} < 10J$ for the particles; and
determining packing density by $\phi_{wjI} = \phi_M - \sum \phi_{wj}$ when $u_{jI} < 10J$ and permeation flux is established by inertial lift for more than one particle type.

35. The method according to claim 31, wherein said determining packing density further comprises:

determining if permeation flux is established by J_I for more than one particle type ($jI1, jI2, \dots, jIn$); and
determining packing density at the membrane wall by
 $\phi_{wjI1} + \phi_{wjI2} = \phi_M - \sum \phi_{wj}$, wherein $\phi_{wjI1} : \phi_{wjI2} = \phi_{bjI1} u_{ji2} : \phi_{bjI2} u_{ji1}$,
where $j \neq jI1$ or $jI2$ and $u_{ji1}, u_{ji2} < 10J$, when permeation flux is established by J_I for more than one particle type.

36. The method according to claim 30, wherein said determining interstitial packing density ($\phi_{wiinterstice}$) of the smallest particle is carried out by

$\phi_{wiinterstice} = \phi_{wicorrected} / (1 - \sum \phi_{wjcorrected})$, wherein $\phi_{wicorrected} = \phi_M [(\phi_{wi}) / \sum \phi_{wi}]$,
where ϕ_{wi} is the particle volume fraction at the membrane wall (-) for particle i .

37. The method according to claim 31 further comprising:
re-calculating packing density for all particle sizes and

determining if packing constraints are not satisfied based on initial determination of packing densities of the particles at the wall.

38. The method according to claim 37 further comprising:
correcting packing density by using $\phi_{w\text{corrected}} = \phi_M [(\phi_{wi})/\Sigma \phi_{wi}]$;
reevaluating J for the particle selected as having the minimum permeation flux based on $\phi_{w\text{corrected}} = \phi_M [(\phi_{wi})/\Sigma \phi_{wi}]$; and
reevaluating maximum back-transport velocity (u_i).

39. The method according to claim 30, wherein filtration is selected from the group consisting of microfiltration and ultrafiltration.

40. A method for predicting pressure independent permeation flux for crossflow membrane filtration of a poly-disperse suspension, said method comprising:

determining viscosity of the suspension;
determining maximum back-transport velocity (u_i) for all particles;
estimating maximum aggregate packing volume fraction (ϕ_M) for all particles at a wall of the filtration membrane from geometric considerations;
selecting the particle that gives a minimum permeation flux at a given filtration membrane shear rate, wherein the selected particle has a radius (a_i);

determining a predicted permeation flux (J); and
determining packing density (ϕ_{wj}) at the membrane wall for each particle size (a_j for $j \neq i$) at the predicted permeation flux, thereby predicting pressure independent permeation flux for the suspension.

41. The method according to claim 40 further comprising:
re-calculating packing density for all particle sizes if packing constraints are not satisfied based on initial determination of packing densities at the wall.

42. The method according to claim 41 further comprising:

correcting packing density using $\phi_{w\text{corrected}} = \phi_M [(\phi_{wi})/\Sigma$

$\phi_{wi}]$;

reevaluating J for the particle selected as having the minimum permeation flux based on $\phi_{w\text{corrected}} = \phi_M [(\phi_{wi})/\Sigma \phi_{wi}]$; and

reevaluating maximum back-transport velocity (u_i).

43. The method according to claim 40, wherein said

determining viscosity of the suspension is carried out by using a modified

Einstein-Smoluchowski equation: $\eta/\eta_0 = 1 + 2.5\phi_b + k_1\phi_b^2$, where η is bulk fluid viscosity (kg/m.s) of the suspension, η_0 is bulk fluid viscosity of the suspension without solute (kg/m.s), k_1 is particle shape factor (-), and ϕ_b is particle volume fraction in the bulk suspension.

44. The method according to claim 40, wherein said

determining viscosity of the suspension is carried out by experiment.

45. The method according to claim 40, wherein said

determining maximum back-transport velocity (u_i) comprises:

calculating Brownian diffusion (J_B) for all particles, where

$$J_B = 0.114(\gamma\kappa^2 T^2/\eta^2 a^2 L)^{1/3} \ln(\phi_w/\phi_b);$$

calculating inertial lift (J_I) for all particles, where $J_I =$

$$0.036\rho a^3 \gamma^2/\eta;$$

calculating shear induced diffusion (J_S) for all particles, where $J_S =$

$0.078(a^4/L)^{1/3} \gamma \ln(\phi_w/\phi_b)$, and wherein γ is wall shear rate (s^{-1}), κ is Boltzmann constant ($J/mol K$), T is temperature (K), η is bulk fluid viscosity (kg/m.s), a_i is radius of species i (m), L is tube length (m), ϕ_w is particle volume fraction at the membrane wall (-), ϕ_b is the particle volume fraction in the bulk suspension (-), and ρ is particle density (kg/m^3); and

selecting J_{max} for each particle, wherein $J_{\text{max}} = u_i$, thereby determining maximum back-transport for each particle.

46. The method according to claim 40, wherein said estimating maximum aggregate packing volume fraction(ϕ_M) at the membrane wall for a suspension comprises:

determining particle size (a_i) of species (i) in the suspension;

determining if the size ratio of the particles is > 10 , such that $a_{i+1} > 10a_i$ for all a_i ; and

calculating the maximum aggregate packing volume fraction(ϕ_{Mn})

by

$\phi_{Mn} = \phi_m + \phi_m (1 - \phi_{Mn-1})$, where $\phi_M = \phi_m$ set to 0.64, when the size ratio the particles is > 10 , such that $a_{i+1} > 10a_i$ for all a_i .

47. The method according to claim 40, wherein the suspension comprises 3 particle sizes and wherein $a_1 > 10a_2 > 100a_3$, said method further comprising:

calculating $\phi_M = \phi_m + \phi_m (1 - \phi_m) + 0.74[1 - \{\phi_m + \phi_m (1 - \phi_m)\}]$,

wherein ϕ_m is the maximum packing volume fraction for monodisperse spheres set to 0.64. .

48. The method according to claim 40, wherein said estimating maximum aggregate packing volume fraction (ϕ_M) at the membrane wall comprises:

calculating a maximum radius ratio of all particles;

determining if said maximum radius ratio is < 10 ; and

setting ϕ_M as 0.68, where said maximum radius ratio is < 10 .

49. The method according to claim 40, wherein said estimating maximum aggregate packing volume fraction (ϕ_M) at the membrane wall for a suspension comprising two particles, such that $a_1 > 10 a_2$, is carried out by calculating $\phi_M = \phi_m + 0.74 (1 - \phi_m)$, where ϕ_m is set to 0.64.

50. The method according to claim 40, wherein said selecting the particle that gives a minimum permeation flux (J) comprises:

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calculating Brownian diffusion (J_B) for all particles, where

$$J_B = 0.114(\gamma\kappa^2T^2/\eta^2a^2L)^{1/3}\ln(\phi_w/\phi_b);$$

calculating inertial lift (J_I) for all particles, where $J_I =$

$$0.036\rho a^3\gamma^2/\eta;$$

calculating shear induced diffusion (J_S) for all particles, where $J_S = 0.078(a^4/L)^{1/3}\gamma\ln(\phi_w/\phi_b)$, wherein γ is wall shear rate (s^{-1}), κ is Boltzmann constant ($J/mol\ K$), T is temperature (K), η is bulk fluid viscosity ($kg/m.s$), a_i is radius of species i (m), L is tube length (m), ϕ_w is particle volume fraction at the membrane wall (-), ϕ_b is the particle volume fraction in the bulk suspension (-), and ρ is particle density (kg/m^3);

determining a J_{max} value for each particle; and

selecting a J_{max} value from among all J_{max} values that is the lowest, thereby selecting the minimum permeation flux (J).

51. The method according to claim 40, wherein said determining packing density at the membrane wall (ϕ_{wj}) for all particles at the predicted permeation flux (a_j for $j \neq i$) comprises:

back-calculating the value of ϕ_{wj} such that ϕ_{wj} gives the predicted permeation flux (J) of selected particle (a_i), using the equation for back-transport that establishes maximum back transport for each particle (a_j for $j=i$), wherein the equation is either $J_B = 0.114(\gamma\kappa^2T^2/\eta^2a^2L)^{1/3}\ln(\phi_w/\phi_b)$ or $J_S = 0.078(a^4/L)^{1/3}\gamma\ln(\phi_w/\phi_b)$, or $J_I = 0.036\rho a^3\gamma^2/\eta$, where γ is wall shear rate (s^{-1}), κ is Boltzmann constant ($J/mol\ K$), T is temperature (K), η is bulk fluid viscosity ($kg/m.s$), a_i is radius of species i (m), L is tube length (m), ϕ_w is particle volume fraction at the membrane wall (-), ϕ_b is the particle volume fraction in the bulk suspension (-), and ρ is particle density (kg/m^3).

52. The method according to claim 51, wherein said determining packing density further comprises:

determining if the predicted permeation flux is established by inertial lift (J_I) for one particle type;

determining if $u_{f1} \geq 10J$; and

setting $\phi_{wj} = 0$, when one particle type is established by inertial lift (J_I) and $u_{jI} \geq 10J_I$.

53. The method according to claim 51, wherein said determining packing density further comprises:

- determining if the predicted permeation flux is established by inertial lift (J_I) for one particle type;
- determining if $u_{jI} < 10J$; and
- determining packing density (ϕ_{wj}) by $\phi_{wjI} = \phi_M - \sum \phi_{wj}$ when $u_{jI} < 10J$ and one particle type is established by inertial lift.

54. The method according to claim 51, wherein said determining packing density further comprises:

- determining if permeation flux is established by inertial lift (J_I) for more than one particle type;
- determining if $u_{jI} < 10J$ for the particles; and
- determining packing density by $\phi_{wjI} = \phi_M - \sum \phi_{wj}$ when $u_{jI} < 10J$ and permeation flux is established by inertial lift for more than one particle type.

55. The method according to claim 51, wherein said determining packing density further comprises:

- determining if permeation flux is established by J_I for more than one particle type ($jI1, jI2, \dots, jIn$); and
- determining packing density at the membrane wall by $\phi_{wjI1} + \phi_{wjI2} = \phi_M - \sum \phi_{wj}$, wherein $\phi_{wjI1} : \phi_{wjI2} = \phi_{bjI1} u_{jI1} : \phi_{bjI2} u_{jI1}$, where $j \neq jI1$ or $jI2$ and $u_{jI1}, u_{jI2} < 10J$, when permeation flux is established by J_I for more than one particle type.

56. The method according to claim 40, wherein filtration is selected from the group consisting of microfiltration and ultrafiltration.

57. A method for calculating yield of a target molecule in a permeate for a poly-disperse suspension during crossflow membrane filtration, said method comprising:

determining minimum pore diameter ($2r_{\text{minimum}}$) based on the packing density of each particle and

estimating yield of a target species in the filtration permeate by calculating observed sieving coefficient (S_o) for the target species.

58. The method according to claim 57, further comprising:
refining the yield and pressure independent permeation flux.

59. The method according to claim 57, wherein said refining the yield comprises:

determining whether the suspension has a low, intermediate, or high operating shear rate leading to different yield regimes, wherein a suspension at a low operating shear rate leads to an $S_o \geq 0.75$ corresponding to a yield ≥ 0.95 , an intermediate operating shear rate leads to $0 < S_o < 0.75$ corresponding to yield from 0 to 95%, or a high operating shear rate leads to an $S_o \cong 0$, wherein $S_o = S_a / ((1 - S_a) \exp(-J/k) + S_a)$, wherein actual sieving coefficient S_a is obtained from $S_a = (S_\infty \exp(Pe_m)) / (S_\infty + \exp(Pe_m) - 1)$, wall Peclet number, Pe_m is obtained from $Pe_m = (J \delta_m / D) (S_\infty / \varepsilon \phi K_d)$, where J is permeation flux (m/s), δ_m is taken as the side of the face centered cube of the particles of radius a_i that forms the controlling cake for transmission, where $\delta_m = a = a_i [(4(4/3)\pi) / \phi_{\text{interstice}}]^{1/3}$, D is molecular diffusion coefficient (m^2/s), intrinsic sieving coefficient S_∞ is obtained from $S_\infty = (1 - \lambda)^2 [2 - (1 - \lambda)^2] \exp(-0.7146\lambda^2)$, $\lambda = r_s / r_{\text{min}}$, where r_s is solute radius (m) and r_{min} is a minimum equivalent cake void radius for all cake types (m), ϕ is equilibrium partition coefficient between membrane pore and suspension (-), ε is cake/membrane porosity (-), K_d is hindrance factor for diffusive transport (-), and k is mass transfer coefficient (m/s).

60. The method according to claim 59, wherein an intermediate operating shear rate is determined as leading to $0 < S_o < 0.75$, said method further comprising:

calculating stagnant film flux (J) equation for non-retentive membranes wherein $J = k \ln [(\phi_{wi} - \phi_{permeatei}) / (\phi_{bi} - \phi_{permeatei})] \cong k \ln [\phi_{wi} / \phi_{bi}(1 - S_o)]$, wherein $(\phi_{wi} \gg \phi_{permeatei})$; and

correcting S_o by replacing $J =$ solvent permeation flux (m/s) with the stagnant film flux (J) equation for non-retentive membranes in the equation for observing sieving coefficient, S_o , where $S_o = S_a / ((1 - S_a) \exp(-J/k) + S_a)$.

61. The method according to claim 57, wherein determining minimum pore diameter ($2r_{\text{minimum}}$) is carried out using

$2r_{\text{minimum}} = a_i \{ \sqrt{2[4(4/3)\pi / \phi_{wiinterstice}]^{1/3} - 2} \}$, where a is the radius of species i (m) and r_{minimum} is a minimum equivalent cake void radius for all cake types (m).

62. The method according to claim 57, wherein said estimating yield of a target species comprises:

calculating observed sieving coefficient (S_o), where $S_o = S_a / ((1 - S_a) \exp(-J/k) + S_a)$, wherein actual sieving coefficient S_a is obtained from $S_a = (S_\infty \exp(Pe_m)) / (S_\infty + \exp(Pe_m) - 1)$, wall Peclet number, Pe_m is obtained from $Pe_m = (J \delta_m / D)(S_\infty / \varepsilon \phi K_d)$, where J is permeation flux (m/s), δ_m is taken as the side of the face centered cube of the particles of radius a_i that forms the controlling cake for transmission, where $\delta_m = a = a_i [(4(4/3)\pi) / \phi_{iinterstice}]^{1/3}$, D is molecular diffusion coefficient (m^2/s), intrinsic sieving coefficient S_∞ is obtained from $S_\infty = (1 - \lambda)^2 [2 - (1 - \lambda)^2] \exp(-0.7146\lambda^2)$, $\lambda = r_s / r_{\text{min}}$, where r_s is solute radius (m) and r_{min} is a minimum equivalent cake void radius for all cake types (m), ϕ is equilibrium partition coefficient between membrane pore and suspension (-), ε is cake/membrane porosity (-), K_d is hindrance factor for diffusive transport (-), and k is mass transfer coefficient (m/s).

63. The method according to claim 57, wherein crossflow filtration is carried out in a diafiltration mode, said the yield of the target species after N_d diavolumes is estimated by $\text{Yield} = 1 - \exp(-N_d S_{\text{average}})$, where S_{average} is average observed sieving coefficient during diafiltration (-), where $S_o = S_d / ((1 - S_d) \exp(-J/k) + S_d)$, where actual sieving coefficient S_d is obtained from $S_d = (S_\infty \exp(Pe_m)) / (S_\infty + \exp(Pe_m) - 1)$, wall Peclet number, Pe_m , is obtained from $Pe_m = (J \delta_m / D) (S_\infty / \varepsilon \phi K_d)$, where J is permeation flux (m/s), δ_m is taken as the side of the face centered cube of the particles of radius a_i that forms the controlling cake for transmission, where $\delta_m = a = a_i [(4(4/3)\pi) / \phi_{\text{interstice}}]^{1/3}$, D is molecular diffusion coefficient (m^2/s), intrinsic sieving coefficient S_∞ is obtained from $S_\infty = (1 - \lambda)^2 [2 - (1 - \lambda)^2] \exp(-0.7146 \lambda^2)$, $\lambda = r_s / r_{\text{min}}$, where r_s is solute radius (m) and r_{min} is a minimum equivalent cake void radius for all cake types (m), and ϕ is equilibrium partition coefficient between membrane pore and suspension (-), ε is cake/membrane porosity (-), K_d is hindrance factor for diffusive transport (-), and k is mass transfer coefficient (m/s).

64. The method according to claim 57, wherein filtration is selected from the group consisting of microfiltration and ultrafiltration.

65. The method according to claim 57, wherein filtration is carried out with a filter selected from the group consisting of a flat sheet filter, hollow-fiber filter, and a helical filter.

66. The method according to claim 57, wherein the suspension is selected from the group consisting of streams from biomedical and bio-processing industries, waste water, surface water, environmental pollutants, industrial waste streams, and industrial feed streams.

67. The method according to claim 66, wherein the suspension is a stream from biomedical and bio-processing industries selected from the group consisting of proteins, cells, nucleic acids, colloids, milk, and suspended particles.

68. A method for designing a crossflow membrane filtration system for a poly-disperse suspension, said method comprising:

selecting a poly-disperse suspension;

applying the method according to claim 1 to predict pressure independent permeation flux and target molecule yield in a permeate for the selected poly-disperse suspension; and

optimizing conditions for filtration based on the prediction of permeation flux and target molecule yield to design a filtration system for the selected poly-disperse suspension.

69. The method according to claim 68, wherein filtration is selected from the group consisting of microfiltration and ultrafiltration.

70. The method according to claim 68, wherein filtration is carried out with a filter selected from the group consisting of a flat sheet filter, hollow-fiber filter, and a helical filter.

71. A method of selecting operating conditions of a crossflow filtration system for poly-disperse suspensions, said method comprising:

applying the method of claim 1 to a crossflow filtration system for a selected poly-disperse suspension to determine a limiting pressure independent permeation flux for a given shear rate and expected yield of a target species for the selected system conditions and

selecting the operating conditions of the system using the determined limiting pressure independent permeation flux for a given shear rate to obtain an optimal balance between permeation flux and yield of a target species.

72. The method according to claim 71, wherein filtration is selected from the group consisting of microfiltration and ultrafiltration.

73. The method according to claim 71, wherein filtration is carried out with a filter selected from the group consisting of a flat sheet filter, hollow-fiber filter, and a helical filter.

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74. The method according to claim 71, wherein the suspension is selected from the group consisting of waste water, surface water, environmental pollutants, industrial waste streams, and industrial feed streams.

75. A method of modeling a process for filtration of a poly-disperse suspension comprising:

applying the method according to claim 1 for a poly-disperse suspension using a computer-generated program to model a process for filtration of the poly-disperse suspension.

76. The method according to claim 75, wherein filtration is selected from the group consisting of microfiltration and ultrafiltration.

77. The method according to claim 75, wherein filtration is carried out with a filter selected from the group consisting of a flat sheet filter, hollow-fiber filter, and a helical filter.

78. The method according to claim 75, wherein the suspension is selected from the group consisting of streams from biomedical and bio-processing industries, waste water, surface water, environmental pollutants, industrial waste streams, and industrial feed streams.

79. The method according to claim 78, wherein the suspension is a stream from biomedical and bio-processing industries selected from the group consisting of proteins, cells, nucleic acids, colloids, milk, and suspended particles.